Int. J. Heat Mass Transfer. Vol. 16, pp. 1371–1384. Pergamon Press 1973. Printed in Great Britain

HIGH REYNOLDS NUMBERS UNSTEADY CONVECTIVE MASS TRANSFER FROM FLUID SPHERES

VI-DUONG DANG
New York University, New York, New York, U.S.A.
and
ELI RUCKENSTEIN†
Department of Chemical Engineering, University of Delaware, Newark, Delaware 19711, U.S.A.

(*Received* 15 *May* 1972 *and in revised form* 1 *November* 1972)

Abstract—Unsteady state mass transfer between a single or binary component fluid sphere and the continuous phase at high Reynolds number flow is examined when diffusion is accompanied (or not) by a first order chemical reaction. The velocity distribution derived by Chao is used in the calculations. The similarity transformation suggested by Ruckenstein is applied to find the mass transfer in the absence of chemical reactions from a single component fluid sphere. For mass transfer with or without chemical reactions from a binary fluid sphere, the same transformation is combined with Duhamel's theorem in order to obtain the solution.

Asymptotic expressions for the Sherwood number for pure mass transfer from a single component fluid sphere and for the case in which diffusion is accompanied by a first order irreversible chemical reaction are derived. For binary component fluid spheres the quasi steady state assumption (QSSA) is examined and its results compared with the exact analysis.

NOMENCLATURE

a, radius of the bubble;
b, defined by equation (5);
B, $\dfrac{4b}{3\sqrt{(\pi Re)}}$;
C, concentration of the diffusing species;
C_d, concentration of the dispersed phase;
C_{d0}, initial concentration of the dispersed phase;
C^*, equilibrium concentration in the continuous phase;
D, diffusion coefficient;
H, distribution coefficient;
k, first order irreversible reaction rate constant;
K, overall mass transfer coefficient;
N, average mass transfer rate;
N_θ, local mass transfer rate;
Pe, $\dfrac{3aU_\infty}{2D}$;
R, $\dfrac{ka^2}{D}$;
Re, $\dfrac{2U_\infty a\rho_1}{\mu_1}$;
Sh, $\dfrac{2Ka}{D}$;
r, radius coordinate of the system measured from the center of the drop;
t, time;
$u(t)$, $\dfrac{\omega_2(t) - H\omega_\infty}{1 - H\omega_\infty}$;
U_∞ translational velocity of the center of the drop;
v_r, radial velocity component;
v_θ, tangential velocity component;
X, argument of the arbitrary function ϕ in the left hand side of equation (16);
y, $r-a$;
Y, y/a.

Greek symbols
α_1, α_2, parameters;

† To whom correspondence should be sent.

β, positive constant;
δ, length introduced by means of the similarity variable η;
η, Y/δ;
μ, viscosity;
ρ, density;
ν, kinematic viscosity;
ζ, defined by equation (4);
τ, tD/a^2;
τ', $Pe\,\tau$;
θ, polar coordinate;
θ_s, separation angle;
Δ, δ^2;
ϕ, arbitrary function;
ϕ_1, defined by equation (19a);
ω, C/C_{d0} for binary component bubble; C/C^* for single component bubble.

Subscripts
1, continuous phase;
2, dispersed phase;
∞, bulk liquid condition.

INTRODUCTION

MASS transfer between a fluid sphere and the continuous phase at high Reynolds numbers flow was first solved by Boussinesq [2] who obtained the steady state expression for the Sherwood number assuming potential flow. By using a different technique Ruckenstein [11] has solved the unsteady state case. Levich [9], superimposing a boundary layer upon the potential flow, has shown that for sufficiently large Reynolds numbers the potential flow represents a good approximation of the velocity field. Chao [3, 14] has obtained on the basis of Levich's procedure a corrected velocity distribution which is valid in a larger range of Reynolds numbers than the potential distribution. Winnikow [13] obtained the steady state Sherwood number at large Reynolds and Schmidt numbers. Cheh and Tobias [5] studied the same problem independently.

In many processes such as mass transfer from binary (or multi-component) bubbles or drops to the continuous phase, the concentration of the diffusing species in the dispersed phase is time dependent. Therefore it is necessary to consider the unsteady state transfer in these processes. The authors studied this problem in references [6, 12] for low Reynolds numbers and potential flow.

The aim of the present paper is to examine the unsteady state mass transfer in a higher range of Reynolds numbers by using the more realistic equations obtained by Chao [3] for the velocity distribution. The following situations will be considered:

(1) Unsteady mass transfer in the continuous phase from a single component bubble or drop.

(2) (a) Unsteady mass transfer between a binary bubble or drop and the continuous phase.
(b) Validity of the quasi-steady state assumption.

(3) (a) Unsteady state mass transfer from a single or binary component fluid sphere to the continuous phase when diffusion is accompanied in the continuous phase by a first order irreversible chemical reaction.
(b) Validity of the quasi steady state assumption for the binary component case.

The problems are solved by using the similarity transformation suggested previously [11].

I. UNSTEADY MASS TRANSFER FROM A SINGLE COMPONENT BUBBLE OR DROP

Consider a single component bubble or drop moving in a liquid phase. A diffusing species is transferred to the continuous phase such as in the gas absorption process. It is assumed that the bubble keeps its spherical shape and constant size; the flow is axisymmetric; thermodynamic equilibrium exists at the fluid–fluid interface. Therefore one can write the unsteady convective diffusion equations in spherical coordinates for the continuous phase as

$$\frac{\partial C}{\partial t} + v_r \frac{\partial C}{\partial r} + \frac{v_\theta}{r}\frac{\partial C}{\partial \theta} = D\left[\frac{\partial^2 C}{\partial r^2} + \frac{2}{r}\frac{\partial C}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial C}{\partial \theta}\right)\right]. \quad (1)$$

The velocity components of the continuous phase given by Chao [3, 14] for high Reynolds numbers flow are

$$\frac{v_r}{U_\infty} = -\left(1 - \frac{a^3}{r^3}\right)\cos\theta + \frac{4b}{3Re}\left\{\left[\frac{1}{2} + \left(\frac{1-\cos\theta}{\sin^2\theta}\right)^2\right]\operatorname{erf}\zeta + 2\left[1 - \left(\frac{1-\cos\theta}{\sin^2\theta}\right)^2\right]\zeta\,\operatorname{ierfc}\zeta\right\} \quad (2)$$

$$\frac{v_\theta}{U_\infty} = \left(1 + \frac{a^3}{2r^3}\right)\sin\theta - \frac{2b(\sqrt{3})}{\sqrt{(Re)}}\frac{(\frac{2}{3} - \cos\theta + \frac{1}{3}\cos^3\theta)^{\frac{1}{2}}}{\sin\theta}\operatorname{ierfc}\zeta \quad (3)$$

where

$$\zeta = \frac{\sqrt{(3Re)}}{4}\frac{\sin^2\theta}{(\frac{2}{3} - \cos\theta + \frac{1}{3}\cos^3\theta)^{\frac{1}{2}}}\frac{y}{a} \quad (4)$$

$$b = \frac{2 + 3(\mu_2/\mu_1)}{1 + (\rho_2\mu_2/\rho_1\mu_1)^{\frac{1}{2}}} \quad (5)$$

and

$$Re = \frac{2U_\infty a \rho_1}{\mu_1}, \qquad y = r - a.$$

For high Reynolds and Schmidt numbers, the diffusion boundary layer is thin. Consequently only the velocity distribution close to the fluid–fluid interface is of importance. Expanding the velocity components in terms of the distance y from the fluid–fluid interface and retaining only the first terms, one obtains

$$\frac{v_r}{U_\infty} = \frac{3}{2}\left[-\cos\theta + \frac{b}{\sqrt{(3\pi\, Re)}}\frac{\sin^2\theta}{(\frac{2}{3} - \cos\theta + \frac{1}{3}\cos^3\theta)^{\frac{1}{2}}}\right]\frac{2y}{a} \quad (6)$$

$$\frac{v_\theta}{U_\infty} = \frac{3}{2}\left[\sin\theta - \frac{4b}{\sqrt{(3\pi\, Re)}}\times\frac{(\frac{2}{3} - \cos\theta + \frac{1}{3}\cos^3\theta)^{\frac{1}{2}}}{\sin\theta}\right]. \quad (7)$$

In equation (1), molecular diffusion in the θ direction may be neglected. Because in the region of interest $y \ll a$, the term $(v_\theta/r)(\partial C/\partial\theta)$ may be approximated by $(v_\theta/a)(\partial C/\partial\theta)$. By introducing the dimensionless quantities

$$\tau = \frac{tD}{a^2}, \qquad Y = \frac{y}{a} = \frac{r-a}{a}, \qquad Pe = \frac{3aU_\infty}{2D}$$

and substituting equations (6) and (7) into equation (1), the convective diffusion equation can be written in the following dimensionless form:

$$\frac{\partial C}{\partial \tau} + 2\,Pe\left[-\cos\theta + \frac{b}{\sqrt{(3\pi\, Re)}}\times\frac{\sin^2\theta}{(\frac{2}{3} - \cos\theta + \frac{1}{3}\cos^3\theta)^{\frac{1}{2}}}\right]Y\frac{\partial C}{\partial Y} + Pe\left[\sin\theta - \frac{4b}{\sqrt{(3\pi\, Re)}}\times\frac{(\frac{2}{3} - \cos\theta + \frac{1}{3}\cos^3\theta)^{\frac{1}{2}}}{\sin\theta}\right]\times\frac{\partial C}{\partial\theta} = \frac{\partial^2 C}{\partial Y^2}. \quad (8)$$

The initial and boundary conditions are

$$C(0, Y, \theta) = C_\infty \quad (9)$$

$$C(\tau, \infty, \theta) = C_\infty \quad (10)$$

$$C(\tau, 0, \theta) = C^* \quad (11)$$

C^* is the equilibrium concentration at the interface. It is possible to write the solution of equations (8)–(11) in the following form [11]:

$$C(\tau, Y, \theta) = C(\eta) \quad (12)$$

where

$$\eta = \frac{Y}{\delta(\tau, \theta)}. \quad (13)$$

With these transformations, one can obtain the concentration distribution in the continuous phase as (see Appendix)

$$\frac{C - C_\infty}{C^* - C_\infty} = \operatorname{erfc} \eta. \tag{14}$$

The quantity $\delta(\tau, \theta)$ is given by

$$\delta(\tau, \theta) = \frac{2}{\sqrt{(Pe)}(1 - \cos\theta)\left[1 + \cos\theta - B\sqrt{(2 + \cos\theta)}\right]} \Bigg\{ \tfrac{1}{3}\cos^3\theta - \cos\theta + 2B\left[(2 + \cos\theta)^{\frac{3}{2}} - \frac{(2 + \cos\theta)^{\frac{5}{2}}}{5}\right] + \phi\Bigg[Pe\,\tau + \frac{2}{4 - 3B^2}\ln\left(\frac{1 + \cos\theta - B\sqrt{(2 + \cos\theta)}}{1 - \cos\theta}\right) + \frac{4B}{(4 - 3B^2)\sqrt{(4 + B^2)}}\ln\left(\frac{2\sqrt{(2 + \cos\theta)} - B - \sqrt{(4 + B^2)}}{2\sqrt{(2 + \cos\theta)} - B + \sqrt{(4 + B^2)}}\right) + \frac{B\sqrt{(3)}}{4 - 3B^2} \times \ln\left(\frac{\sqrt{(3)} + \sqrt{(2 + \cos\theta)}}{\sqrt{(3)} - \sqrt{(2 + \cos\theta)}}\right)\Bigg]\Bigg\}^{\frac{1}{2}} \tag{15†}$$

where

$$B \equiv \frac{4b}{3\sqrt{(\pi\,Re)}}$$

and the function ϕ is defined by

$$\phi\left[\frac{2}{4 - 3B^2}\ln\left(\frac{1 + \cos\theta - B\sqrt{(2 + \cos\theta)}}{1 - \cos\theta}\right) + \frac{4B}{(4 - 3B^2)\sqrt{(4 + B^2)}} \times \ln\left(\frac{2\sqrt{(2 + \cos\theta)} - B - \sqrt{(4 + B^2)}}{2\sqrt{(2 + \cos\theta)} - B + \sqrt{(4 + B^2)}}\right) + \frac{B\sqrt{(3)}}{4 - 3B^2}\ln\left(\frac{\sqrt{(3)} + \sqrt{(2 + \cos\theta)}}{\sqrt{(3)} - \sqrt{(2 + \cos\theta)}}\right)\right]$$
$$= \cos\theta - \tfrac{1}{3}\cos^3\theta - 2B\left[(2 + \cos\theta)^{\frac{3}{2}} - \frac{(2 + \cos\theta)^{\frac{5}{2}}}{5}\right]. \tag{16}$$

The local flux of the diffusing specie is

$$N_\theta = -D\frac{\partial C}{\partial y}\bigg|_{y=0} = \frac{2(C^* - C_\infty)D}{\sqrt{(\pi)}a\,\delta} \tag{17}$$

and ignoring the mass transfer in the wake, the average mass transfer rate is given by

$$N = \frac{1}{4\pi a^2}\int_0^{\theta_s} 2\pi a^2 N_\theta \sin\theta\, d\theta$$
$$= \frac{(C^* - C_\infty)D}{a\sqrt{(\pi)}}\int_0^{\theta_s}\frac{\sin\theta\, d\theta}{\delta(\tau, \theta)} \tag{18}$$

where θ_s is the angle of separation. Information concerning the values of θ_s is available in [14].

If one defines the mass transfer coefficient K as

$$K = \frac{N}{C^* - C_\infty},$$

then the Sherwood number is given by

$$Sh = \frac{2Ka}{D} = \left(\frac{Pe}{\pi}\right)^{\frac{1}{2}}\int_0^{\theta_s}\frac{(1 - \cos\theta)(1 + \cos\theta - B\sqrt{(2 + \cos\theta)}\sin\theta\, d\theta}{\left\{\tfrac{1}{3}\cos^3\theta - \cos\theta + 2B\left[(2 + \cos\theta)^{\frac{3}{2}} - \frac{(2 + \cos\theta)^{\frac{5}{2}}}{5}\right] + \phi_1(Pe\,\tau, \theta)\right\}^{\frac{1}{2}}} \tag{19}$$

where

† In equations (15) and (16) $\phi[\ ]$ represents ϕ function of the argument inside the brackets.

$$\phi_1(Pe\,\tau, \theta) \equiv \phi\left[Pe\,\tau + \frac{2}{4 - 3B^2}\ln\left(\frac{1 + \cos\theta - B\sqrt{(2 + \cos\theta)}}{1 - \cos\theta}\right) + \frac{4B}{(4 - 3B^2)\sqrt{(4 + B^2)}}\right.$$
$$\left.\times \ln\left(\frac{2\sqrt{(2 + \cos\theta)} - B - \sqrt{(4 + B^2)}}{2\sqrt{(2 + \cos\theta)} - B + \sqrt{(4 + B^2)}}\right) + \frac{B\sqrt{(3)}}{4 - 3B^2}\ln\left(\frac{\sqrt{(3)} + \sqrt{(2 + \cos\theta)}}{\sqrt{(3)} - \sqrt{(2 + \cos\theta)}}\right)\right]. \quad (19a)$$

When $B = 0$, equation (16) leads to

$$\phi\left(-\ln\tan\frac{\theta}{2}\right) = \cos\theta - \frac{1}{3}\cos^3\theta. \quad (20)$$

By means of the trigonometric identity

$$\cos\theta = \frac{1 - \tan^2(\theta/2)}{1 + \tan^2(\theta/2)} = \frac{1 - \exp[2\ln\tan(\theta/2)]}{1 + \exp[2\ln\tan(\theta/2)]}$$

one can then obtain

$$\phi\left(Pe\,\tau - \ln\tan\frac{\theta}{2}\right) = \frac{1 - \exp[2(-Pe\,\tau + \ln\tan(\theta/2)]}{1 + \exp[2(-Pe\,\tau + \ln\tan(\theta/2)]} - \frac{1}{3}\left\{\frac{1 - \exp[2(-Pe\,\tau + \ln\tan(\theta/2)]}{1 + \exp[2(-Pe\,\tau + \ln\tan(\theta/2)]}\right\}^3 \quad (21)$$

This is the result obtained previously by Ruckenstein [11] for potential flow. Substituting equation (21) into equations (15) and (19) with $B = 0$, one can obtain the same expressions for δ and Sh as in [11].

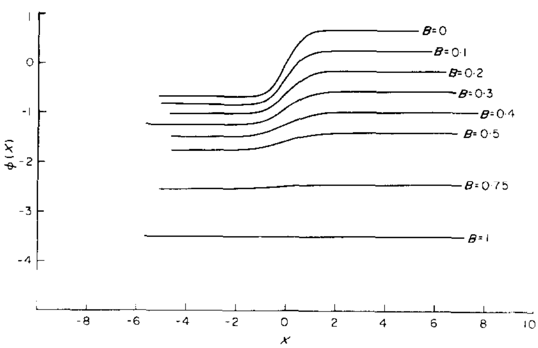

FIG. 1. Graphical determination of the arbitrary function ϕ.

For $B \neq 0$ no explicit expression can be established for ϕ and numerical procedures are necessary. If we denote the argument of the arbitrary function ϕ inside the bracket in equation (16) as X and then evaluate X and $\phi(X)$ at different values of the angle θ, we can represent $\phi(X)$ versus X graphically. The result of this calculation is shown in Fig. 1. In general, ϕ changes more rapidly in the range $-1 \leqslant X \leqslant 1$ and attains asymptotic values at $X > 1$ and $X < -1$. As $B \gtrsim 0.75$, the value of ϕ is practically constant and the variation of ϕ is less than about 4 per cent. From Fig. 1, one can obtain an asymptotic expression for $\phi(\infty)$ as

$$\phi(\infty) = \frac{2}{3} - \frac{12B\sqrt{(3)}}{5} \quad (22)$$

A comparison of the numerical values of $\phi(\infty)$ from equation (16) and those from equation (22) is shown in Table 1. One may observe that equation (22) is a very good approximation for the steady state values of ϕ. Therefore one can

Table 1. Comparisons of the values of ϕ at large argument between the numerical values from equation (16) and asymptotic values from equation (22)

B	$\phi(\infty)$ equation (16)	$\phi(\infty)$ equation (22)
0	0·667	0·667
0·1	0·251	0·251
0·2	−0·165	−0·165
0·3	−0·580	−0·581
0·4	−0·996	−0·997
0·5	−1·412	−1·410
0·75	−2·451	−2·450
1	−3·490	−3·490

obtain a steady state expression for δ by substituting equation (22) into equation (15). This leads to

$$\delta(\theta) = \frac{2\left\{\frac{2}{3} + \frac{1}{3}\cos^3\theta - \cos\theta + 2B\left[(2+\cos\theta)^{\frac{3}{2}} - \frac{(2+\cos\theta)^{\frac{5}{2}}}{5} - \frac{6\sqrt{(3)}}{5}\right]\right\}^{\frac{1}{2}}}{\sqrt{(Pe)}\,(1-\cos\theta)\,(1+\cos\theta - B\sqrt{(2+\cos\theta)}}. \tag{23}$$

From equations (23) and (14), one obtains

$$\frac{C(Y,\theta) - C_\infty}{C^* - C_\infty} = \operatorname{erfc}\left\{\frac{(1-\cos\theta)\,[1+\cos\theta - B\sqrt{(2+\cos\theta)}]\,Y}{2\left\{\frac{1}{Pe}\left[\frac{2}{3} + \frac{1}{3}\cos^3\theta - \cos\theta + 2B\left((2+\cos\theta)^{\frac{3}{2}} - \frac{(2+\cos\theta)^{\frac{5}{2}}}{5} - \frac{6\sqrt{(3)}}{5}\right)\right]\right\}^{\frac{1}{2}}}\right\}. \tag{24}$$

This reduces to the result obtained by Winnikow [13] who has treated the steady state case. The Sherwood number from equation (19) was computed numerically and the results are given in Fig. 2. One can obtain a simple expression for the steady state Sherwood number by cross plotting the steady state Sherwood number vs B from Fig. 2. This gives

$$\left(\frac{\pi}{Pe}\right)^{\frac{1}{2}} Sh = \frac{4}{\sqrt{(3)}} - 2{\cdot}268B. \tag{25}$$

In Table 2 a comparison is made between the steady state dimensionless Sherwood number calculated from the present analysis, equation (19), with the asymptotic expression equation (25) and with Winnikow's results [13]. The agreement is very good. Cheh and Tobias [5] used the velocity profiles of Moore [10] to study the steady state mass transfer problem and presented their results in graphical form. Both for gas–liquid ($\mu_1 \gg \mu_2$, $\rho_1 \gg \rho_2$) and liquid–liquid ($\mu_1 \approx \mu_2$, $\rho_1 \approx \rho_2$) systems the Sherwood number calculated from the present analysis and that of Cheh and Tobias give practically the same values for $Re \geqslant 80$.

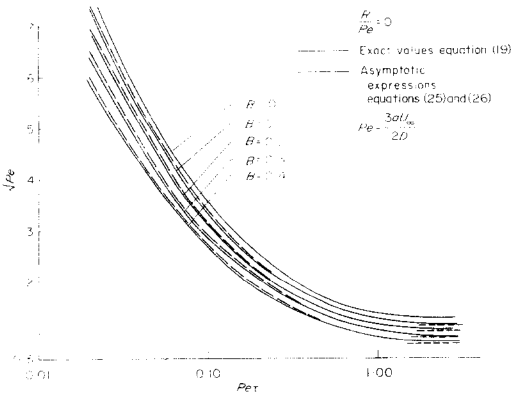

FIG. 2. Sherwood number vs $Pe\,\tau$ for single component bubble for various values of $B(R/Pe = 0)$.

Table 2. Comparison of steady state Sherwood numbers of the present analysis and that of Winnikow [13]

B	Winnikow	Present analysis, equation (19)	Asymptotic equation (25)
0	2·309	2·308	2·309
0·1	2·078	2·085	2·082
0·2	1·858	1·859	1·855
0·3	1·604	1·630	1·629
0·4	1·366	1·399	1·402
0·5	1·145	1·170	1·175

For small times the integrals with respect to θ may be replaced by

$$\frac{1-\cos\theta_s}{\sqrt{(Pe\,\tau)}}$$

and equation (19) reduces to

$$Sh = \left(\frac{1}{\pi\tau}\right)^{\frac{1}{2}}(1 - \cos\theta_s). \qquad (26)$$

Figure 2 shows the dimensionless Sherwood number from equation (19) as a function of $Pe\,\tau$. The error of using the potential flow for mass transfer process increases with B and slightly with $Pe\,\tau$. For example when $B = 0{\cdot}1$, the errors of using potential flow ($B = 0$) in calculating the dimensionless Sherwood number are 5·1 per cent at $Pe\,\tau = 0{\cdot}01$ and 10·67 per cent at $Pe\,\tau = 7$ respectively. It is found that the deviation of the steady state results from potential flow is less than about 2·3 per cent when $B \gtrsim 0{\cdot}023$.

The value of $Pe\,\tau$ required for the dimensionless Sherwood number to reach the steady state increases slightly as a function of B. When $B = 0$ and 0·4, the steady state dimensionless Sherwood numbers are achieved at $Pe\,\tau = 1{\cdot}5$ and 2 respectively. As B increases, it takes longer time for the dimensionless Sherwood number to attain steady state. For $B \gtrsim 0{\cdot}75$, the time in which the steady state is achieved is not longer than $Pe\,\tau = 3$.

Also compared in Fig. 2 are the asymptotic expressions for the dimensionless Sherwood number, equations (25), (26), with the exact one, equation (19). In general, equations (25) and (26) give good agreement with the exact analysis and therefore can be used for design purposes.

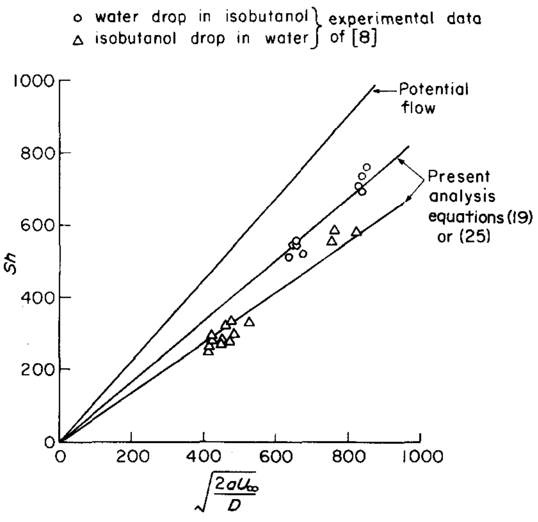

FIG. 3. Comparison of steady state Sherwood number from the present analysis with experimental data in [8].

Figure 3 shows a comparison of the steady state Sherwood number based on the present calculations with some existing experimental data. It is seen that the steady state results from the present analysis compares quite well with the experimental data of Heertjes *et al.* [8].

II. UNSTEADY MASS TRANSFER FROM A BINARY COMPONENT BUBBLE OR DROP

The physical system considered here is similar to the previous one and we assume that the rate determining step is the mass transfer in the continuous phase. For a single component bubble, the concentration of the dispersed phase is constant. However, the concentration of the dispersed phase of a binary bubble is a function of time. The convective diffusion equation, initial and boundary conditions are the same as equations (8)–(10) but the interfacial boundary condition becomes

$$C(\tau, 0, \theta) = \frac{C_2(\tau)}{H} \qquad (27)$$

where H is the distribution coefficient and $C_2(\tau)$ is an unknown function of time which can be determined by means of a material balance over the bubble. This is

$$\frac{2}{3}\frac{d\omega_2(\tau)}{d\tau} = \int_0^{\theta_s} \left.\frac{\partial\omega}{\partial Y}\right|_{Y=0} \sin\theta \, d\theta \qquad (28)$$

where $\omega(\tau, Y, \theta) = C(\tau, Y, \theta)/C_{d0}$, $\omega_2 = C_2/C_{d0}$ and C_{d0} is the initial concentration of the dispersed phase.

The problem is solved by combining the similarity transformation method [11, 12] with Duhamel's theorem [1].

Applying this theorem, one can write

$$\omega(\tau, Y, \theta) = \frac{\partial}{\partial\tau}\int_0^{\tau} F(\tau - \lambda, Y, \theta, \lambda)\, d\lambda, \qquad (29)$$

where F satisfies the same set of equations as ω does except the interface boundary condition is considered time independent by replacing τ by a parameter λ. Therefore F can be readily obtained by the same way as in Part I. Hence the concentration distribution for the continuous phase is

$$\omega(\tau, Y, \theta) = \frac{\partial}{\partial \tau}\int_0^{\tau}\left[\omega_\infty + \left(\frac{\omega_2(\lambda)}{H} - \omega_\infty\right) \times \operatorname{erfc}\frac{Y}{\delta(\theta, \tau - \lambda)}\right]\mathrm{d}\lambda. \tag{30}$$

However $\omega_2(\tau)$ is still an unknown function of time and can be determined by means of equation (28). It is found that

$$u(\tau') = 1 - \frac{3}{2\sqrt{(\pi)}}\int_0^{\tau'}\frac{u(z)}{H\sqrt{(Pe)}} \times \int_0^{\theta_s}\frac{(1-\cos\theta)\,[1+\cos\theta - B\sqrt{(2+\cos\theta)}]\sin\theta\,\mathrm{d}\theta\,\mathrm{d}z}{\left\{\frac{1}{3}\cos^3\theta - \cos\theta + 2B\left[(2+\cos\theta)^{\frac{3}{2}} - \frac{(2+\cos\theta)^{\frac{5}{2}}}{5}\right] + \phi_1(\tau'-z,\theta)\right\}^{\frac{1}{2}}} \tag{31}$$

where $\phi_1(\tau' - z, \theta)$ is given by equation (19a) and

$$\tau' = Pe\,\tau, \qquad u(t) = \frac{\omega_2(t) - H\omega_\infty}{1 - H\omega_\infty}.$$

This integral equation has been solved numerically by means of successive approximations.

Numerical results concerning equation (31) are shown in Fig. 4. With these results one can obtain the concentration distribution in the continuous phase by means of equation (30). Also in Fig. 4 a comparison is made between equation (31) and the results obtained on the basis of the quasi-steady state assumption (QSSA). The QSSA employs the steady state mass transfer in the continuous phase together with an unsteady state mass balance for the bubble. The bubble concentration from QSSA is given by

$$u(\tau') = \exp\left(\frac{-3Sh\,\tau'}{2HPe}\right), \tag{32}$$

where Sh can be calculated from equation (25). The validity conditions of QSSA can be obtained qualitatively as follows: From the above discussion (see also Fig. 2), it results that the mass transfer in the continuous phase for the case of a single component bubble approaches steady state when $Pe\,\tau \backsim 1$. Consequently the time needed to achieve steady state is given by $t \backsim a^2/PeD$. The time required for the bubble concentration to change appreciably can be obtained from equation (32) and is of the order of $\backsim a^2H/D\sqrt{(Pe}$. The quasi-steady state assumption is valid if the second time is larger than the first. Therefore equation (32) is valid if $H\sqrt{(Pe)} \gg 1$.

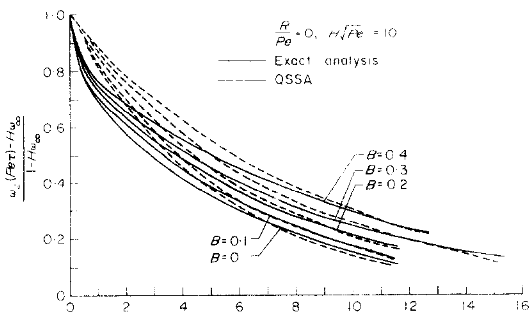

FIG. 4. Bubble concentration vs dimensionless time. Comparison between exact analysis and QSSA.

III. UNSTEADY MASS TRANSFER FROM A SINGLE OR BINARY COMPONENT BUBBLE WITH CHEMICAL REACTIONS

In this case, one assumes that diffusion in the continuous phase is accompanied by a chemical reaction of the first order. With the same assumptions as before, the governing equations for the concentration distribution in the continuous phase are

$$\frac{\partial \omega}{\partial \tau} + 2\,Pe\left[-\cos\theta + \frac{\sqrt{(3)}B}{4} \times \frac{\sin^2\theta}{(\frac{2}{3} - \cos\theta + \frac{1}{3}\cos^3\theta)^{\frac{1}{2}}}\right] Y \frac{\partial \omega}{\partial Y} + Pe\left[\sin\theta - \sqrt{(3)}B\frac{(\frac{2}{3} - \cos\theta + \frac{1}{3}\cos^3\theta)^{\frac{1}{2}}}{\sin\theta}\right] \times \frac{\partial \omega}{\partial \theta} = \frac{\partial^2 \omega}{\partial Y^2} - R\omega \tag{33}$$

$$\omega(0, Y, \theta) = \omega_\infty \tag{34}$$

$$\omega(\tau, \infty, \theta) = \omega_\infty e^{-R\tau} \tag{35}$$

$$\omega(\tau, 0, \theta) = \omega^*(\tau) \tag{36}$$

where $R \equiv ka^2/D$, and k is the rate constant for the first order irreversible reaction. Equation (35) states that the concentration in the bulk of the liquid phase decreases as a function of time due to the chemical reaction. Equation (36) is a thermodynamic equilibrium relation. $\omega^*(\tau)$ is a constant for single component bubbles and a function of time (which can be obtained by means of equation (28)) for binary component bubbles. For binary component bubbles, the assumption is made that the rate determining step of the process is in the continuous phase.

To solve equations (33)–(36), one first introduces the transformation

$$\omega(\tau, Y, \theta) = v(\tau, Y, \theta)\, e^{-R\tau}. \tag{37}$$

This transformation can eliminate the chemical reaction term in equations (33) and (35), but it introduces a multiplication factor $e^{R\tau}$ in the right hand side of equation (36). Therefore the defining equation for $v(\tau, Y, \theta)$ is similar to the one treated in Part II and can be solved using the similarity transformation and the Duhamel's theorem.

The concentration distribution for the continuous phase for a single component bubble is then

$$\omega(\tau, Y, \theta) = e^{-R\tau}\frac{\partial}{\partial \tau}\int_0^\tau \left[\omega_\infty + (e^{R\lambda} - \omega_\infty) \times \operatorname{erfc}\frac{Y}{\delta(\theta, \tau, \lambda)}\right] d\lambda \tag{38}$$

where $\delta(\theta, \tau)$ is again given by equation (15).

If one defines the mass transfer coefficient as

$$K = \frac{-\int_0^{\theta_s} 2\pi a^2 D\,(\partial C/\partial y)|_{y=0}\sin\theta\, d\theta}{4\pi\, a^2(C^* - C_\infty\, e^{-kt})}, \tag{39}$$

then Sherwood number is given by

$$Sh = \frac{2Ka}{D} = \frac{e^{-(R/Pe)\tau'}}{1 - \omega_\infty\, e^{(-R/Pe)\tau'}}\sqrt{\left(\frac{Pe}{\pi}\right)} \times \left[(1 - \omega_\infty)\int_0^{\theta_s}\frac{(1 - \cos\theta)\,[1 + \cos\theta - B\sqrt{(2 + \cos\theta)}]\sin\theta\, d\theta}{\left\{\frac{1}{3}\cos^3\theta - \cos\theta + 2B\left[(2 + \cos\theta)^{\frac{3}{2}} - \frac{(2 + \cos\theta)^{\frac{5}{2}}}{5}\right] + \phi_1(\tau', \theta)\right\}^{\frac{1}{2}}} + \frac{R}{Pe}e^{(R/Pe)\tau'}\int_0^{\tau'} e^{-RZ/Pe} dz \times \int_0^{\theta_s}\frac{(1 - \cos\theta)\,[1 + \cos\theta - B\sqrt{(2 + \cos\theta)}]\sin\theta\, d\theta}{\left\{\frac{1}{3}\cos^3\theta - \cos\theta + 2B\left[(2 + \cos\theta)^{\frac{3}{2}} - \frac{(2 + \cos\theta)^{\frac{5}{2}}}{5}\right] + \phi_1(z, \theta)\right\}^{\frac{1}{2}}}\right] \tag{40}$$

where $\tau' = Pe\,\tau$, and ϕ_1 is given by equation (19a).

For the binary component case, one can obtain the concentration distribution in the continuous phase by the same method as for the single component problem. Consequently

$$\omega(\tau, Y, \theta) = e^{-R\tau}\frac{\partial}{\partial\tau}\int_0^\tau\left[\omega_\infty + \left(\frac{\omega_2(\lambda)\,e^{R\lambda}}{H} - \omega_\infty\right)\times \operatorname{erfc}\frac{Y}{\delta(\theta, \tau-\lambda)}\right]d\lambda. \quad (41)$$

However $\omega_2(\tau)$ is still an unknown function of time and can be determined by means of equation (28). One obtains that with $\omega_\infty = 0$

$$\omega_2(\tau') = e^{(-R/Pe)\tau'}\left[1 + \frac{R}{Pe}\int_0^{\tau'} e^{(R/Pe)t}\,\omega_2(t)\,dt - \frac{3}{2\sqrt{(\pi)}}\int_0^{\tau'}\frac{\omega_2(u)\,e^{(R/Pe)u}}{H\sqrt{(Pe)}}\times\int_0^{\theta_s}\frac{(1-\cos\theta)\,[1+\cos\theta - B\sqrt{(2+\cos\theta)}]\sin\theta\,d\theta\,du}{\left\{\frac{1}{3}\cos^3\theta - \cos\theta + 2B\left[(2+\cos\theta)^{\frac{3}{2}} - \frac{(2+\cos\theta)^{\frac{5}{2}}}{5}\right] + \phi_1(\tau'-u, \theta)\right\}^{\frac{1}{2}}}\right]. \quad (42)$$

It is of interest to find simple asymptotic expressions for the Sherwood number. For small times, the integrals with respect to θ in equation (40) may be replaced by $(1-\cos\theta_s)/\sqrt{(Pe\,\tau)}$ and one obtains

$$\frac{Sh}{\sqrt{(Pe)}} = \frac{e^{(-R/Pe)\tau'}}{\sqrt{(\pi)}(1-\omega_\infty\,e^{(-R/Pe)\tau'})}\left[\frac{(1-\omega_\infty)}{\sqrt{(\tau')}} + \frac{R}{Pe}e^{(R/Pe)\tau'}\sqrt{\left(\frac{\pi Pe}{R}\right)}\operatorname{erf}\sqrt{\left(\frac{R}{Pe}\tau'\right)}\right]\times(1-\cos\theta_s). \quad (43)$$

When $\tau\to\infty$, the first term inside the bracket of equation (40) approaches zero because the exponential factor $e^{-R\tau'}$ decreases rapidly as time increases. In order to evaluate the remaining double integral from equation (40), the asymptotic expressions for large and small time without chemical reactions, equations (25) and (26), will be used for large and small value of z in the inner integral with respect to θ. To evaluate the outer integral with respect to z, it will be split into two parts. In the first part the integral with respect to θ is substituted by the equation (26), valid for small z. In the second part the integral with respect to θ is substituted by equation (25), valid for large z. The value of z for which the transition from one range to another takes place is evaluated from the equality of the two limiting expressions

$$\frac{1-\cos\theta_s}{\sqrt{(Pe\,\tau_s)}} = \frac{4}{\sqrt{(3)}} - 2{\cdot}268B\left(\text{or } Pe\,\tau_s = \left(\frac{1-\cos\theta_s}{4\sqrt{(3)} - 2{\cdot}268B}\right)^2\right).$$

$Pe\,\tau_s$ indicates the transition value from one range to the other.

The first integral is carried out from $z = 0$ to $z = Pe\,\tau_s$ and the second from $z = Pe\,\tau_s$ to τ'. In this manner we can simplify equation (40) and obtain:

$$\frac{Sh}{\sqrt{(Pe)}} = \sqrt{\left(\frac{R}{Pe}\right)}(1-\cos\theta_s)\times\operatorname{erf}\left[\left(\frac{1-\cos\theta_s}{(4/\sqrt{3}) - 2{\cdot}268B}\right)\sqrt{\frac{R}{Pe}}\right] + \frac{1}{\sqrt{(\pi)}}\left(\frac{4}{\sqrt{(3)}} - 2{\cdot}268B\right)\exp\left[-\frac{R}{Pe}\right.$$

$$\times \left(\frac{1 - \cos\theta_s}{(4/\sqrt{3}) - 2{\cdot}268B}\right)^2\Bigg]. \qquad (44)$$

Equations (40), (43) and (44) are plotted in Figs. 2, 5 and 6 for $(R/Pe) = 0$, 1 and 100 with B as parameter. One may observe that the steady state is reached at $\tau_{cs} \backsim (1/Pe)$ when $(R/Pe) \ll 1$ and at $Pe\,\tau_{cs} \backsim (Pe/R)$ when $(R/Pe) \gg 1$. For example, when $(R/Pe) = 0$ (Fig. 2), steady state is attained at $\tau_{cs} \backsim (1/Pe)$, and when $(R/Pe) = 100$ (Fig. 6), steady state is attained at $Pe\,\tau_{cs} \backsim 10^{-2}$. Comparison of equations (43) and (44) with equation (40) generally gives a good agreement. The deviation of the Sherwood number from the values for potential flow can be seen in Figs. 2, 5 and 6. Sherwood number decreases as B increases. As can be seen from Figs. 2, 5 and 6, the reaction parameter, (R/Pe), does not appear to affect the deviation from the potential flow.

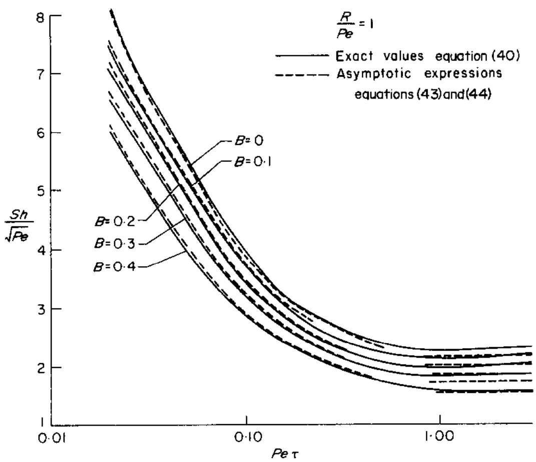

FIG. 5. Sherwood number vs $Pe\,\tau$ for single component bubble for various values of $B(R/Pe = 1)$.

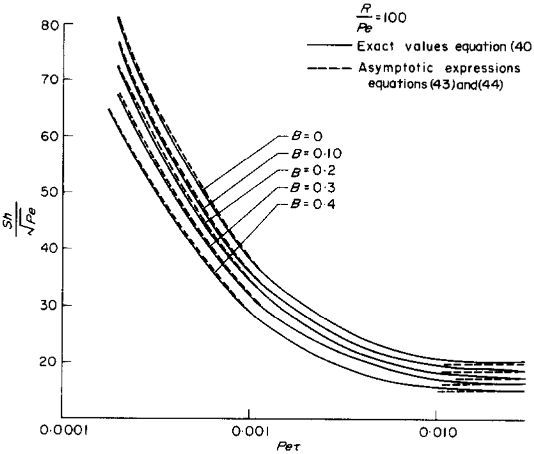

FIG. 6. Sherwood number vs $Pe\,\tau$ for single component bubble for various values of $B(R/Pe = 10^2)$.

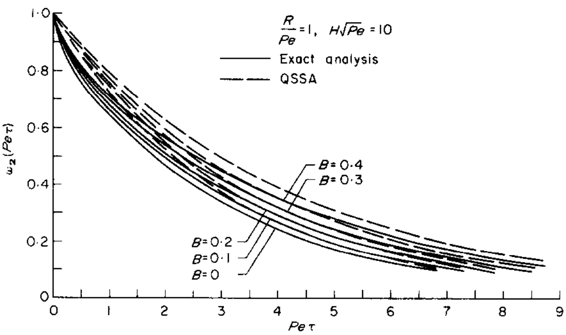

FIG. 7. Bubble concentration vs dimensionless time. Comparison between exact analysis and QSSA.

Solutions of equation (42) for binary component bubbles and the results based upon the quasi steady state approximation (QSSA) are shown in Figs. 4, 7 and 8. In this case the bubble concentration for QSSA is

$$\omega_2(\tau) = \exp\left(\frac{-3\,Sh\,\tau'}{2HPe}\right)$$

where Sh is given by equation (44).

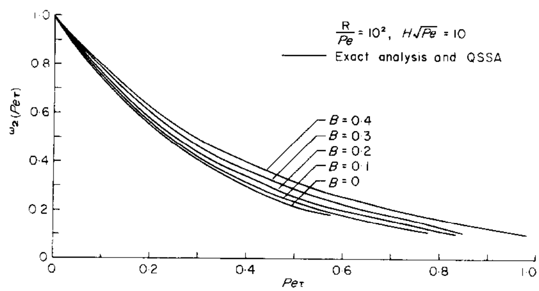

FIG. 8. Bubble concentration vs dimensionless time. Comparison between exact analysis and QSSA.

The criteria for the validity of QSSA can again be obtained by comparing the time in which the concentration of the dispersed phase decreases appreciably with the time in which the continuous phase achieves steady state. This results in the following criteria:

$$H\sqrt{(Pe)} \gg 1 \qquad \text{if} \frac{R}{Pe} \ll 1$$

and

$$H\sqrt{(R)} \gg 1 \qquad \text{if} \frac{R}{Pe} \gg 1.$$

Numerical results show that these criteria are qualitatively satisfactory. When $(R/Pe) = 100$ (Fig. 8), the QSSA and the exact analysis are practically indistinguishable because $H\sqrt{(R)} = 100$. In general, the error of using QSSA appears to be of the same order of magnitude for all values of B.

CONCLUSIONS

(1) Unsteady concentration distribution and the mass transfer rate from a single component bubble or drop have been obtained for relatively large Reynolds numbers (equations (14) and (19)). It is found that the error in the calculation of the Sherwood number by using potential flow is practically negligible when $B < 0{\cdot}02$.

(2) Concentration fields of both the continuous and the dispersed phases have been obtained for mass transfer from a binary component bubble or drop to the continuous phase at high Reynolds number flow assuming the rate determining step in the continuous phase. A qualitative criteria for the validity of the quasi steady state assumption is established.

(3) Solutions for mass transfer from a single or binary component bubble or drop with first order irreversible chemical reactions have been obtained. Asymptotic expressions for the Sherwood number for single component bubble at small time and at steady state are established (equations (43) and (44)). The validity of the quasi steady state assumption for binary component bubble is examined.

REFERENCES

1. C. F. BARATELLO and R. V. CHURCHILL, *Bull. Math. Soc.* **48**, 276 (1942).
2. J. BOUSINESQ, Calcul du pouvoir refroidassant des courants fluides, *J. Math. Pure Appl.* **6**, 285 (1905).
3. B. T. CHAO, Motions of spherical gas bubbles in a viscous liquid at large Reynolds numbers, *Physics Fluids* **5**, 69 (1962).
4. B. T. CHAO, Transient heat or mass transfer to a translating droplet, *J. Heat Transfer* **91**, 273 (1969).
5. H. Y. CHEH and C. W. TOBIAS, Mass transfer to spherical drops or bubbles at high Reynolds number, *I/EC Fundamentals* **7**, 48 (1968).
6. VI. D. DANG, Mass transfer between a fluid sphere and the continuous phase with chemical reactions, Ph.D. Dissertation, Clarkson College of Technology (May 1971).
7. VI. D. DANG and W. N. GILL, Mass transfer across a non-equilibrium interface, *A.I.Ch.E. Jl* **16**, 793 (1970).
8. P. M. HEERTJES, W. A. HOLVE and H. TALSMA, Mass transfer between isobutanol and water in a spray column, *Chem. Engng Sci.* **3**, 122 (1954).
9. V. G. LEVICH, *Physicochemical Hydrodynamics*. Prentice-Hall, New Jersey (1962).
10. D. W. MOORE, The boundary layer on a spherical gas bubble, *J. Fluid Mech.* **16**, 161 (1963).
11. E. RUCKENSTEIN, Mass transfer between a single drop and a continuous phase, *Int. J. Heat Mass Transfer* **10**, 1785 (1967).
12. E. RUCKENSTEIN, VI. D. DANG and W. N. GILL, Mass transfer with chemical reaction from spherical one or two component bubble or drops, *Chem. Engng Sci.* **26**, 647 (1971).
13. S. WINNIKOW, The heat and mass transfer from a fluid sphere at large Reynolds and Peclet numbers, *Can. J. Chem. Engng* **46**, 217 (1968).
14. S. WINNIKOW and B. T. CHAO, Droplet motion in purified systems, *Physics Fluids* **9**, 50 (1966).

APPENDIX

The similarity variable (13) transforms equation (8) into:

$$\frac{d^2C}{d\eta^2} + \eta\frac{dC}{d\eta}\left\{\frac{1}{2}\frac{\partial\delta^2}{\partial\tau} - 2Pe\,\delta^2\left[-\cos\theta + \frac{b}{\sqrt{(3\pi\,Re)}}\frac{\sin^2\theta}{(\frac{2}{3} - \cos\theta + \frac{1}{3}\cos^3\theta)^{\frac{1}{2}}}\right] + \frac{Pe}{2}\left[\sin\theta - \frac{4b}{\sqrt{(3\pi\,Re)}} \times \frac{(\frac{2}{3} - \cos\theta + \frac{1}{3}\cos^3\theta)^{\frac{1}{2}}}{\sin\theta}\right]\frac{\partial\delta^2}{\partial\theta}\right\} = 0. \qquad (A.1)$$

In order for a similarity solution to exist, it is necessary to have

$$\frac{d^2C}{d\eta^2} + \beta\eta \frac{dC}{d\eta} = 0 \qquad (A.2)$$

and

$$\frac{\partial \delta^2}{\partial \tau} - 4Pe\left[-\cos\theta + \frac{b}{\sqrt{(3\pi\, Re)}} \frac{\sin^2\theta}{(\frac{2}{3} - \cos\theta + \cos^3\theta)^{\frac{1}{2}}}\right]\delta^2 + Pe\left[\sin\theta - \frac{4b}{\sqrt{(3\pi\, Re)}} \frac{(\frac{2}{3} - \cos\theta + \frac{1}{3}\cos^3\theta)^{\frac{1}{2}}}{\sin\theta}\right] \times \frac{\partial \delta^2}{\partial \theta} = 2\beta \qquad (A.3)$$

where β is a positive constant and is chosen 2 for convenience. The boundary conditions (9)–(11) then become

$$C(0) = C^* \qquad (A.4)$$

$$C(\infty) = C_\infty. \qquad (A.5)$$

Equation (A.3) is solved by the method of characteristics. Hence

$$\frac{d\tau}{1} = \frac{d\theta}{Pe\left[\sin\theta - \frac{4b}{\sqrt{(3\pi\, Re)}} \frac{(\frac{2}{3} - \cos\theta + \frac{1}{3}\cos^3\theta)^{\frac{1}{2}}}{\sin\theta}\right]} = \frac{d\Delta}{4 + 4Pe\left[-\cos\theta + \frac{b}{\sqrt{(3\pi\, Re)}} \frac{\sin^2\theta}{(\frac{2}{3} - \cos\theta + \frac{1}{3}\cos^3\theta)^{\frac{1}{2}}}\right]\Delta} \qquad (A.6)$$

where $\Delta = \delta^2$.

From the first equation, one gets

$$Pe\,\tau + \frac{2}{4 - 3B^2}\ln\left[1 + \cos\theta - B\sqrt{(2 + \cos\theta)}\right] + \frac{4B}{(4 - 3B^2)\sqrt{(4 + B^2)}} \times \ln\left(\frac{2\sqrt{(2 + \cos\theta)} - B - \sqrt{(4 + B^2)}}{2\sqrt{(2 + \cos\theta)} - B + \sqrt{(4 + B^2)}}\right) - \frac{2}{4 - 3B^2}\ln(1 - \cos\theta) + \frac{\sqrt{(3)}B}{4 - 3B^2} \ln\left(\frac{\sqrt{(3)} + \sqrt{(2 + \cos\theta)}}{\sqrt{(3)} - \sqrt{(2 + \cos\theta)}}\right) = \alpha_1. \qquad (A.7)$$

From the last equation, one gets

$$\Delta(1 - \cos\theta)^2\left[1 + \cos\theta - B\sqrt{(2 + \cos\theta)}\right]^2 = \frac{4}{Pe} \times \left\{\tfrac{1}{3}\cos^3\theta - \cos\theta + 2B\left[(2 + \cos\theta)^{\frac{3}{2}} - \frac{(2 + \cos\theta)^{\frac{5}{2}}}{5}\right] + \alpha_2\,.\right\} \qquad (A.8)$$

Therefore

$$\Delta(1 - \cos\theta)^2\left[1 + \cos\theta - B\sqrt{(2 + \cos\theta)}\right]^2 - \frac{4}{Pe}\left\{\tfrac{1}{3}\cos^3\theta - \cos\theta + 2B\left[(2 + \cos\theta)^{\frac{3}{2}} - \frac{(2 + \cos\theta)^{\frac{5}{2}}}{5}\right]\right\} = \frac{4}{Pe}$$

$$\times\, \phi\left[Pe\,\tau + \frac{2}{4 - 3B^2}\ln\left(\frac{1 + \cos\theta - B\sqrt{(2 + \cos\theta)}}{1 - \cos\theta}\right) + \frac{4B}{(4 - 3B^2)\sqrt{(4 + B^2)}} \times \ln\left(\frac{2\sqrt{(2 + \cos\theta)} - B - \sqrt{(4 + B^2)}}{2\sqrt{(2 + \cos\theta)} - B + \sqrt{(4 + B^2)}}\right) + \frac{\sqrt{(3)}B}{4 - 3B^2} \times \ln\left(\frac{\sqrt{(3)} + \sqrt{(2 + \cos\theta)}}{\sqrt{(3)} - \sqrt{(2 + \cos\theta)}}\right)\right]. \qquad (A.9)$$

The boundary condition

$$\tau = 0, \qquad \Delta = 0 \qquad (A.10)$$

allows one to determine the form of the arbitrary function ϕ (see equation (16)).

CONVECTION MASSIQUE NON PERMANENTE À PARTIR DE SPHERES FLUIDES POUR DES GRANDS NOMBRES DE REYNOLDS

Résumé—On considère le transfert de masse non permanent entre une sphère fluide à composant unique ou binaire et l'écoulement à phase continue pour un grand nombre de Reynolds quand la diffusion est ou n'est pas accompagnée d'une réaction chimique de premier ordre. On utilise dans les calculs la distribution de vitesse de Chao. La transformation de similitude suggèrée par Ruckenstein est appliquée au calcul du transfert massique en l'absence de réactions chimiques depuis une sphère fluide à un seul composant. Quant au transfert de masse avec ou sans réaction chimique depuis une sphère fluide à composant binaire, on applique la même transformation associée au théorème de Duhamel pour la recherche de la solution.

On obtient des expressions asymptotiques pour le nombre de Sherwood relatif à un transfert massique pur depuis une sphère fluide à un composant unique dans le cas où la diffusion s'accompagne d'une réaction chimique irréversible de premier ordre. Pour des sphères fluides à composant binaire l'hypothèse d'état quasi-statique ("QSSA") est étudiée et ses résultats sont comparés à l'analyse exacte.

INSTATIONÄRER KONVEKTIVER STOFFÜBERGANG VON FLÜSSIGKEITSKUGELN BEI HOHEN REYNOLDS-ZAHLEN

Zusammenfassung—Es wird untersucht der instationäre Stofftransport zwischen einer Flüssigkeitskugel, bestehend aus einer oder zwei Komponenten, und der umgebenden Phase bei hohen Reynolds-Zahlen, wenn mit der Diffusion eine chemische Reaktion erster Ordnung gekoppelt ist (oder auch nicht). In den Rechnungen wurde die Geschwindigkeitsverteilung, wie sie von Chao abgeleitet wurde, benutzt. Die von Ruckenstein vorgeschlagene Ähnlichkeitstransformation wurde angewendet, um den Stofftransport ohne chemische Reaktionen von einer Einkomponenten-Flüssigkeitskugel zu bestimmen. Für den Stoffübergang mit oder ohne chemische Reaktionen von einer Zweikomponenten-Flüssigkeitskugel wurde die selbe Transformation kombiniert mit dem Theorem von Duhamel angewendet, um zur Lösung zu kommen.

Es sind asymptotische Ausdrücke abgeleitet für die Sherwood-Zahl für reinen Stoffübergang von einer Einkomponenten-Flüssigkeitskugel und für den Fall mit gleichzeitiger irreversibler chemischer Reaktion erster Ordnung. Für die Zweikomponenten-Flüssigkeitskugeln wurde die Annahme von quasistationärem Ablauf überprüft und deren Ergebnisse mit der exakten Untersuchung verglichen.

НЕСТАЦИОНАРНЫЙ КОНВЕКТИВНЫЙ ПЕРЕНОС МАССЫ ОТ СФЕРИЧЕСКИХ ЧАСТИЦ ЖИДКОСТИ ПРИ БОЛЬШИХ ЗНАЧЕНИЯХ ЧИСЛА РЕЙНОЛЬДСА

Аннотация—Исследуется нестационарный массообмен между сферической частицей одно-или двухкомпонентной жидкости и непрерывной фазой при больших значениях числа Рейнольдса потока, когда диффузия сопровождается или не сопровождается химической реакцией первого порядка. В расчётах используется распределение скорости, полученное Чао. Для определения переноса массы от сферической частицы однокомпонентной жидкости при отсутствии химических реакций используется преобразование подобия, предложенное Рукенштейном. Для получения решения в случае переноса массы от сферической частицы двухкомпонентной жидкости при наличии или отсутствии химических реакций используется аналогичное преобразование в сочетании с теоремой Дюамеля.

Получены асимптотические выражения для числа Шервуда в случае чистого переноса массы от сферической частицы однокомпонентной жидкости и в случае, когда диффузия сопровождается необратимой химической реакцией первого порядка. Для сферических частиц двухкомпонентной жидкости рассматривается предположение о квазистационарности, а полученные результаты сравниваются с результатами точного анализа.